

# Two Mathematical Spaces, One Roof: The Local and Nonlocal Structures of the Unit Circle

## 1. Interpreting $\sqrt{-1}$

The unit circle is the infinitely symmetrical structure of the two-dimensional plane. Within this space there are two descriptions, and each is fundamental. The familiar and classically-based description is that in which the orthogonal directions are the  $x$ - and  $y$ -axes. The second is that in which the  $y$ -axis is rather  $iy$ . The transformation that occurs when  $y$  becomes  $iy$  is dramatic. Since the term  $i$  (standing for  $\sqrt{-1}$ ) is not a rational construction, neither is the space in which it defines orthogonality. Thus, although the same geometric structure (the unit circle) contains two descriptions, these descriptions are not the same. This is the same feature that defines the Russell set called  $R$ .<sup>1</sup>

The key feature of  $R$  is that it is not a member of itself. Thus, for any structure that mirrors the property of  $R$ , we will not be able to state that the parts of this structure are common in the given domain created by them, and this is paradoxical. In IToE, the presence of paradox in fundamental relationships is considered authentic and decipherable. No attempt is made to explain paradox away. Rather it is shown that paradox is a natural mechanism.

The unit circle can be described using the *complex unit circle* (the Wessel-Argand-Gauss plane). The  $y$ -axis becomes the  $iy$ -axis. The property of all locations on the vertical orthogonal axis are thrown into a state of confusion, in rational terms, for the classical observer when  $i$  is incorporated. This confusion is reflected in the mathematical definition of  $i$  which is as follows:

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<sup>1</sup> See <http://www.pathcom.com/~dougill/1.2.pdf>, The Paradoxical Reversal of Property for discussion of the Russell set.

$$\text{If } a > 0, \sqrt{-a} = i\sqrt{a}; \quad i^2 = -1 \quad (1.1)$$

$$\text{Then } (\sqrt{-2})^2 = (\sqrt{-2})(\sqrt{-2}) = (i\sqrt{2})(i\sqrt{2}) = (i^2)(2) = -2 \quad (1.2)$$

$$\text{Thus, } (\sqrt{-2})^2 = (\sqrt{-2})(\sqrt{-2}) \neq \sqrt{[(-2)(-2)]} = \sqrt{4} = 2 \quad (1.3)$$

We are not permitted to draw rational conclusion as to the relationship of parts within the square-root sign. This is reflected in the fact that these terms cannot be operated on as common and separate elements within the square-root sign. Rather, for any operation that is performed on this structure, the term  $i$  must first be removed (isolated). Then the portion  $i$  remains a nonrational factor, and the squaring of  $i$  does not produce a plus sign for the resulting value of  $i$ . We see how the above operational procedure differs when the values within the square roots are positive and can be collected. For  $(\sqrt{2})^2$ ,

$$(\sqrt{2})(\sqrt{2}) = \sqrt{[(2)(2)]} = \sqrt{4} = 2 \quad (1.4)$$

<p>(a)</p> $(\sqrt{-2})(\sqrt{-2})$ <p style="text-align: center;">↑</p> <p>[operable relationship]</p>	<p>(b)</p> $\sqrt{[(-2)(-2)]}$ <p style="text-align: center;">↑</p> <p>[regrouping not allowed]</p>
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**Illustration 1.** The regrouping shown in (b) is not allowed. This preserves the nonrational property of the individual terms in (a). That they cannot be members of themselves.

The distinction in Ill. 1 is that the parts in (a) have an “observable” relationship and the parts in (b) do not. The parts are *observable* in the sense that anything *observable* must be rational. The difference between parts that form a common collection and that do not is reflected in the positioning of the observer to the terms. Specifically, (in equation line 1.2) for the operation of collecting terms and multiplying through, the rational portions  $[(2)(2)]$  must be

separated from the nonrational portions  $[(i) (i)]$  so that  $i^2$  is not linked in the manipulation that results in the product (4). In other words, we are not permitted to manipulate two terms that contain a nonrational component - entry into this state of common terms is disallowed.

The is strong similarity between  $R$  and  $(i)$  is that  $\sqrt{-1} = \sqrt{-1} (+1)$ , but  $(-1) (+1)$  are not the same. Thus, although the parts should be the same (have common property), they are not the same (as identified by sign). In other words, even though the parts form a common domain, they are not common.

## 2. The $iy$ -axis as a dualism

For the two-dimensional plane of the unit circle, the set of all locations on the  $y$ -axis is

$$-1 \leq y \leq +1 \tag{2.1}$$

However when  $i$  is appended to all  $y$  as  $iy$ , a state of confusion, in classical terms, is also introduced to the collection of these locations since the value of  $\sqrt{-1}$  cannot be resolved for the parts. This confusion can be shown by applying the meaning of the literal values of  $i$  (if  $i$  where rational) to the vertical axis.

The literal value of  $\sqrt{-1}$  is paradoxically plus and minus one  $[(+1)(-1)]$  not plus or minus one. When applied to the vertical axis of the unit circle this state of confusion is transferred. Specifically, the range of the vertical orthogonal plane becomes,

$$[(+1) \text{ and } (-1)] (-1) \leq iy\text{-axis of the unit circle} \leq [(+1) \text{ and } (-1)] (+1) \tag{2.2}$$

Multiplying  $(-1)$  and  $(+1)$  on each side of  $(\leq iy \leq)$  by  $[(+1) \text{ and } (-1)]$ , the best we can do is distinguish an incorrect and paradoxical ordering such as

$$\begin{array}{ccccccc} -1 \text{ and } +1 & \leq & iy\text{-axis of the unit circle} & \leq & +1 \text{ and } -1 & & (2.3) \\ \text{nonrational} & & \text{and} & & \text{nonrational} & & \end{array}$$

For the left side of Equation (2.3), because the function greater than or equal to applies to

-1 and to +1,  $iy$  is paradoxically both inside and outside of its own range for the unit circle. There is no rational format for the expression in Equation (2.3). A rational perspective can only be created if some intrinsic element of the construction is summarily left out. The plane has open conflicts across the operators and values that apply.

### 3. Analogous paradoxical structures to the $iy$ -axis

The fact that the range of the vertical orthogonal axis is both inside and outside of the unit circle is analogous to the paradoxes found for the Russell set and the conclusion drawn in Cantor's diagonal slash argument. In both these instances, the members of sets defined in one regard are not properly defined in a second regard, and as such, those sets are found to be both inside and outside of their own range.<sup>2</sup>

The construction is also consistent with the conclusion for Gödel's Incompleteness Theorem. Gödel's theorem proves that within an existing system of axioms certain truths can be known to be true; yet, when the axioms are applied to these truths, the truths cannot be proved or disproved.<sup>3</sup> Paradox is the operative mechanism. Considering the left side of Equation (2.3), the true proposition under the axiom or operational rule (in classical terms) is that  $y$  is greater than or equal to  $-1$ . This does not account for the contradiction, on the same side, that  $y$  is also greater than or equal to  $+1$  (known to be true by the internal paradoxical structure of  $i$ ). Our classical bias for the structure of the unit circle only accounts for the first value.

If we interpret this structure literally, then the axiom that defines it applies in a manner that exceeds the limit that it should in a classical description. As in Gödel's conclusion, all the parts of the axiom that are true exceed the limit of the parts that are rational and true.

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<sup>2</sup> See Chapter 1.2, Three Examples of the Reversal of Property for discussion of these two arguments and Gödel's Incompleteness Theorem.

<sup>3</sup> Penrose, 1994: Ch. 2.5.

Although this structure based on  $iy$  is not rational, the legitimate identity of  $iy$  in mathematics is certainly not in question. The resulting structure simply does not make sense under logicism. Plus and minus values are entangled on both the plus and minus sides of the vertical axis. Finally, the  $iy$ -axis contains the fundamental feature of all nonlocal structures that clear and rational locations (in space-time) cannot be identified.

#### 4. The $x$ -axis

A change complementary to the perspective found on the  $iy$ -axis (for the incorporation of paradox and dualism) occurs on the plane of the  $x$ -axis. This change is not recognized in the standard mathematical interpretation of the complex unit circle because the range of the plane of the  $x$ -axis in classical terms is deemed

$$-1 \leq x \leq +1 \quad (4.1)$$

However, this does not take into account that the complementary term to  $i$ , for the  $x$ -axis is  $[\sqrt{+1}x]$  not simply  $(x)$ . Then the  $x$ -axis of the complex unit circle is

$$[\sqrt{+1}](-1) \leq x\text{-axis of the unit circle} \leq [\sqrt{+1}](+1) \quad (4.2)$$

The expanded value of  $\sqrt{+1}$  is  $+1$  or  $-1$ . Grouping the values in Equation (4.2) across or we have

$$\begin{aligned} \text{for } [\sqrt{+1}] &= [+1] \\ [+1](-1) &\leq x\text{-axis of the unit circle} \leq [+1](+1) \quad (\text{rational}) \end{aligned} \quad (4.3)$$

or

$$\begin{aligned} \text{for } [\sqrt{+1}] &= [-1] \\ [-1](-1) &\leq x\text{-axis of the unit circle} \leq [-1](+1) \quad (\text{nonrational}) \end{aligned} \quad (4.4)$$

$$\begin{aligned} \text{Thus,} \quad -1 &\leq \text{domain of } x \leq +1 \quad \text{or} \quad +1 \leq \text{domain of } x \leq -1 \quad (4.5) \\ &\text{rational} \quad \quad \quad \text{or} \quad \quad \quad \text{nonrational} \end{aligned}$$

One half of the stated range for the  $x$ -axis of unit circle is not included in the space we classically recognize as the unit circle, and, in complementary format to the  $iy$ -axis, a paradoxical

construction now exists.

## 5. Analogous paradoxical structures to the $\sqrt{+1}$ $x$ -axis

As for the  $iy$ -axis, the domain of  $x$  within the unit circle contains a component that is not rational from our classical bias. Only the left side across the operator or, in Equation (4.5), is rational. As for the Russell set, each half of the set of  $x$  is not a member of the other half. Thus, the set of all  $x$ , as defining the range of the unit circle for  $x$ , is not a member of itself. As for Cantor's argument, values are found that are not contained in their own limit.

Finally, for Gödel's argument, the nonrational axiom that applies results in a dualism for the absolute limit to which the axioms should apply. For Equation (4.5), the left side of or is a rational axiom, and the right side of or is a nonrational axiom. The axiom for the construction of  $x$  exceeds the infinity established under our rational and classical bias for the limit of the unit circle. Thus, as established by Gödel, all the true propositions of the axiom exceed the limit of what can be established as true under our formal, rational, and classical understanding.

## 6. The important distinction in or: Part of its structure can be summarily ignored

The confusion that exists for the nature of the  $x$ -axis is not apparent in a standard mathematical interpretation of the nonclassical unit circle. The half of the set of values that is rational can be selected, while the other half of the set,  $+1 \leq x \leq -1$ , which is outside of the range of  $x$  and is not rational, can be summarily ignored. The paradoxical dualism, which exists in the full literal interpretation of the space, has been eliminated without obvious consequences. However, this is a simplification that hides the full significance and potential of the properties are contained. The mathematician has taken liberty in dispensing with the alternative range of the  $x$ -axis in which the value  $-1$  is appended to all locations of  $x$  under the expansion of  $\sqrt{+1}$ . This liberty is taken by the classical observer in order to preserve a rational context. However, theory and Nature are telling us that paradox remains as a hidden element.

We see that the  $x$ -axis also contains the fundamental feature identified for nonlocal structures (i.e., that clear, distinct, and rational location cannot be identified). If the parts of a structure are *clear*, *distinct*, and *rational* then there can be no doubt as to the limits and locations that apply. For the  $x$ -axis of the unit circle, the limit of what is rational is exceeded, and a portion is identifiable that is nonlocal.

## 7. Dimensional reconstruction of complex space back to its classical form

The dimensional reconstruction of the complex unit circle back to a classical space is just the reverse of the dimensional reduction of the space and is accomplished by taking the square.

$$\text{For the } x\text{-axis} \quad (\sqrt{+1})^2 x = x \quad \text{and} \quad (7.1)$$

$$\text{For the } iy\text{-axis} \quad (\sqrt{-1})^2 y = -y \quad (7.2)$$

The simple but important point is that a process of dimensional transformation, up and down, applies to both axes across their local and nonlocal interpretations. In the reconstructed classical planes of Equations (7.1) and (7.2), the paradoxes that were opened between the operators and values of locations, which revealed a more complex structure of symmetry within the unit circle, are reclosed.

### 7.1 Loss of information

The dimensional transformation up and down is not reversible for sign in Equation (7.2). The transformation back to the classical form results in the appending of a minus sign to all locations on the  $y$ -axis. The reversal of sign can be viewed as the most fundamental example of loss of information. The information for how the plane should be reconstructed for its sign is lost. The result of the reconstruction is categorically wrong in classical terms.

The emergence of classical structure requires that the more complex description is subsumed into a singular format. The classical version does not contain sufficient distinction of parts to specify the paradoxical properties of the subclassical structure. Consequently the

distinction across its parts is lost, in order to create what is observable in a more dimensionally-complex structure.

The loss of information for the property of the vertical orthogonal axis occurs because the value of the axis is reconstructed by squaring the term  $i$ . In this process  $i$  has been assigned to two locations and a rational operation has been applied across them. In other words, the loss of information points to the fact that the dimensional components have been constructed as existing on a fixed dimensional plane when the correct format for their presentation is one of development across dimensional boundaries.

Specifically,  $[(i) (i)]$  is constructed as if the terms are copies found in a common space. Under the Impressionist Theory of Everything (IToE), this is an inaccurate representation of the process of dimensional development. The level of certainty for the property of the state that exists when the two locations of  $i$  are fully formed on the same plane, is not correct in developmental terms, and compensation is necessary. This compensation takes the form of loss of information.

A loss of information also applies to the  $x$ -axis for the same process of reconstructing the classical space. However, the loss is not apparent because the appropriate values for the locations on the axis can be selected from the alternatives found across or. The lost information is then hidden in what is not selected under the bias of the classical observer.

## **8. The unit circle is, itself, uncertain**

In our classical view of the dimensional structure of the unit circle, the orthogonal planes are copies of direction under rotation. However, uncertainty exists because there are two formats for the construction of these orthogonal axes. Also, for the unit circle an internal confusion exists as to the limit of both orthogonal directions that is based on the difference between the operators or and and respectively. Not only do these two defining directions of the space contain their

separate confusions on limit, but, also, there is a confusion of property between them since the  $x$ - and  $iy$ -axes are not proper copies of direction (the manner in which each is paradoxically constructed is unique).

The symmetry for the terms of confusion established by the relationship of and and or is complementary. For and, it arises because clear alternative is not displayed, and for or, it arises because clear alternative is displayed. In both cases, the paradoxical conclusions that result offend our classical bias. These two formats for the structure of paradox also signify a complex superposition of alternatives. Although the alternatives are contained in the same domain, they are not local members of the domain for their properties. The mechanism which allows the orthogonal directions to be combined in a common space, in spite of the fact that they are nonlocal, is paradox. In any manner that these structures are considered as a dualism, an inconsistency for singular definition arises.

The context in which paradox is incorporated into structure, under the reversal of relationship, begins to take on complexity and detail. Regardless of the observational perspective taken something is left out.

## **9. Paradox is responsible for the mechanism of change**

There are as many forms for the dichotomy of singularism/dualism as there are properties in the universe. In each format, paradox is fundamentally displayed when, for some characteristic that is certain, a second, uncertain characteristic applies for a complete description of Nature. This feature has been described for the identity of the value  $i$ , the range of the  $iy$ -axis, the range of the  $\sqrt{1}x$ -axis, the relationship between the axes as they compose the complex unit circle, and the unit circle itself.

The feature of change that is identified across the certain and uncertain parts of a common domain comes in three fundamental formats. It can be displayed as the static relationship

between alternative and paradoxical descriptions. This is the format generally found in EPR-type experiments and theoretic structures. It can be displayed as the relationship between certain and uncertain values of correlated properties. In this case, when the value of one property is more certain a second property is proportionally uncertain. Finally, change can be displayed as the natural forces found in the classical universe of the observer. Anything that the observer observes in this domain must change under appropriate criteria of space-time (see Chapter 1.7, The Hexorthogonal Geometry of Subclassical Space).

### **10. The reversal of property across subclassical and classical structures**

The technique for representing the interior of an observationally closed space (a quantum-mechanical space of nonlocal parts) is to rearrange the relationships and properties of the objects in this space to be inappropriate (paradoxical) from the classical perspective. The quantum mechanical term for this reversal is *entanglement*. The dimensionally simple model of IToE serves to establish a first-principle for the cross-dimensional relationship of entangled observables and the classical description to which this description refers.

### **11. Quantum mechanical spaces and size**

The term *microscopic* (that infers some function of size is operative) is sometimes applied to describe structure that is quantum mechanical. Under IToE, this label is imprecise. Rather, the distinction between a quantum-mechanical and classical description of a common space is based on the fact that identities and relationships are paradoxical between the two descriptions.

### **12. Conclusion**

For the simplest composition of structure, under the terms of reference of the Impressionist Theory of Everything (IToE), two nonlocal structures are found that are subclassical, and they form a domain that is classically singular. For the interior of this classical singularism the observer has crossed a dimensional boundary to a level that must display paradox

if invariance to the classical level is to be conserved. The problem and error in interpreting the meaning of such a space from a purely classical (observational) perspective is that rationalism does not account for the nature of the alternatives.

This theory takes the concept of relativism into a new realm that distinguishes between perspectives of observation across dimension boundaries. Regardless of the nature or complexity of any structure under consideration, the model of the IToE tells us that whatever is singularly observed with certainty omits a portion of the structure that is paradoxically composed and uncertain. Both of these legitimate components form a larger and more comprehensive description that is not singularly observable.

Physicists, philosophers, and mathematicians are likely to discount this first emergence of paradox precisely because the representations found under formalisms separately based in a classical (Einsteinian) or quantum mechanical formats work flawlessly. However, under this approach, the scholar has headed down the infinitely long and slippery road of trying to explain away paradox by more and more complex structures. The first and clearest opportunity to deal definitively with the issue of paradox, that accounts for it in the most general relationship of local and nonlocal phenomena, has been lost.

February 8, 2004

## REFERENCES

1. Penrose, Roger. 1994. Shadows of the Mind, Oxford: Oxford University Press.