The Paradoxical Reversal of Property in Three Theoretic Structures

1. The reversal of property

The three examples of reversal of property presented in this paper are diverse in their formats, yet a common element emerges to support the central concept of the Impressionist Theory of Everything (IToE): Paradox is an authentic and generic mechanism in the universe. Each instance identifies how some reversal of property plays a key role in the paradoxical structure. The salient characteristics of paradox that are identified are then used to construct a mathematical and geometric model.

The natural presence of paradox is most clearly revealed in the study of properties found within structures that are absolutely fundamental at the classical level. Such domains are infinities in the classical universe. When the interior properties of such infinite spaces are studied, it is found that the defined properties exceed the limit that applies in singular terms. The first characteristic of paradoxical properties so specified is that they are strongly self-referential. A circularity of definition arises that is not rationally controllable.

The important consequence of this paradoxical circularity for the process of observation and rational conclusion is that a juxtaposed dualism of opposing nontransformable structures is always found within the domain that is identified as an absolute classical singularism. The dualism's composition is a larger description than singularly observable. In each example that follows, the reader should note how the reversal of relationship across the objects of the domain is represented and results in a fundamental dualistic perspective on boundary. The mechanism has a unique appearance in each example, but points to the same process.

In order to show that a structure is a rational construction, some formal statement that the structure is a bound state is required as proof. This boundary becomes an infinity condition for the properties of the state. It is at such boundary that paradox is always found. Our descriptions of the universe are very complex and represent the refinement of our understanding of the nature of boundary in infinite structures. However as we approach these absolute limits, the existence of paradox indicates that it is not possible to rationally define such boundary. Each example points to the fact that structures openly displaying infinities are rationally unmanageable for their boundary conditions because the operational relationship across the parts to this dualism is paradox.

2. Cantor's diagonal slash argument

Under Cantor's diagonal slash argument, a problem arises when attempting to define infinity as a theoretic collection of mathematical objects in an appropriate boundary condition. Roger Penrose describes the argument and its boundary problem in his book The Emperor's New Mind. In the words of Penrose, Cantor proved that, " . . . the number of real numbers is actually greater than the number of rational numbers and is not countable."[1]

Cantor used a rational argument to assign the listing of real numbers between zero and one to natural locations. It appeared that a bound and infinite set had been successfully defined, since all possible elements of the set had been included. However, when Cantor used a second axiom (which was the diagonal slash argument) to construct an object that should have been included in the first set, he found that it was not.

More locations were required to specify the collection of real numbers than could be accounted for at the rational limit of infinity as defined in the first part of the argument. Thus, the set of real numbers exceeded the limit of universe (its boundary of infinity), defined as a collection of rational and natural locations.

The nature of the nonresolvable paradox found in Cantor's argument fits with the rationale

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[1] Penrose, 1991: p. 85.

presented in the Impressionist Theory of Everything (IToE). Under IToE, paradox is a natural mechanism in the universe that is most clearly identified when a domain is constructed as infinitely inclusive of its appropriate structure. For the classical universe, such a structure can represent an infinitely small or infinitely large collection of properties. In both cases, an absolute singular perspective of observation has been formed. This singularism is in direct conflict with the fact that any properly structured (properly closed) infinite domain must also include an absolute dualism.

The mechanism that transforms the singularism to the alternative dualism, both of which describe a common domain, is the paradoxical reversal of the relationship between the objects of the common domain for the critical property that applies. Because the dualism is larger (more comprehensive and complex) than singularly observable by a classical observer it limits the inclusiveness of any singular perspective of observation that the observer can perceive. A central theme of IToE is the identification of the specific mechanism of paradox that is at work in each example. Identification of the individual shapes that paradox assumes does not eliminate it but does allow interpretation of its presence in rational terms.

Note: The mathematical/geometric model of IToE shows the cross-dimensional nature of the relationship between domains that are paradoxical but also bound in a common space. These domains exist across a dimensional boundary, and it is this boundary that causes the commonality of the complementary domains to be hidden to the observer. The mechanism for this is paradox. In empirical terms, structure is <u>hidden from observation</u>, and in theoretical terms, structure is <u>hidden from rationalism</u>. In both cases, parts forming a dualism cannot be combined in a common structure. In mathematical terms, there are two ways of constructing this hidden dualism: It can be contained quantum mechanically or noncontained classically.

The manner in which property is fundamentally <u>reversed</u> to create the complementary dualism can be demonstrated in Cantor's argument. In this case, the structure is purely classical, and the dualism is noncontained for the classical space that can be rationally constructed. The infinite collection of the real numbers between 0 and 1 is represented as a vertical column. Neither the ordering of the numbers in the column, nor the complete display of the set is important to the representation. For the example in Illustration 1, the diagonal number in this case is 0.192 . . .

The diagonal number is necessarily a member of the vertical listing since this listing is known to be infinite. This diagonal number is now used to construct a new number using a simple technique that transforms the property of each of the individual digits of the diagonal number and thus of the number itself, into a new number.

Natural position	real value
0	**0**.0 0 0 0 . . .
1	0.**1** 8 4 5 9 2 0 3 4 8 3 9 4 . . .
2	0.3 **9** 5 7 8 2 0 9 4 7 3 8 4 . . .
3	0.5 6 **2** 3 8 2 1 5 2 4 6 7 8 . . .
4	etc.

Illustration 1. Cantor's diagonal slash argument: The digits on the diagonal are used to construct a new number. The digits of this number are then fundamentally altered by reversal of their individual identities.

As an example of this transformation, the property of <u>oneness</u> for the digits of the diagonal number is selected. There are two possible conditions for the property of <u>oneness</u> for each digit on the diagonal: The digit is either <u>one</u> or it is <u>not one</u>.

Now comes the important step of reversing the observable property of the individual digits and the number on the diagonal. The rule for reversal of the property is to transform <u>is</u> to <u>isn't</u>. For this procedure, the second observable property of <u>twoness</u> is used. All <u>1s</u> become <u>not 1s</u> (they become 2s), and all <u>not 1s</u> (2s or otherwise) become <u>1s</u>. Thus, <u>isn't</u> becomes <u>is</u>, and <u>is</u> becomes <u>isn't</u> for the property of <u>oneness</u> in each digit.

The result for the diagonal number 0.192. . . is 0.211. . .

The consequence of this transformation is that the individual digits of the particular number are not members of their own original identity as defined for oneness, and the number that they form on the diagonal is equally not a member of itself.

Finally, this number is not even a member of the original infinite set represented by the vertical column. This can be shown by trying to place the newly created diagonal number in the vertical listing. The new number on the diagonal should be identical to some number (not shown) in the vertical column since the column is an infinite listing. However, this is not the case. Specifically, for the diagonal position in any number listed in the vertical column, the property of the digit is reversed to the property of the same digit in the newly created number.

Regardless of how far down the list this comparison is made, in any horizontal number the same situation will be found for the digit on the diagonal. Thus categorically, the new number cannot be a member of the vertically listed, infinite set. Paradox arises because a set that should be an infinite listing does not include all of its members.

This example illustrates the general principle of the Impressionist Theory of Everything (IToE): When the infinite boundary of any set is represented and an appropriate property of the set is reversed through the mechanism of paradox, the domain beyond the rational boundary of infinity is revealed, completing the structure of an inherent fundamental dualism. The relationship created across the two parts to this dualism is necessarily not rational because the infinite boundary that is rational is smaller than represented by the larger structure of the dualism.

The arbitrarily chosen property in this demonstration was the property of oneness and its relationship to twoness and not twoness, excluding oneness. If sixness and its relationship to twoness and not twoness, excluding sixness had been chosen, we would have found another member of the set that was not included in the vertical listing. The members that are not included in the set under this process of reversal of property are also infinite, and for the full specification of infinity, two fundamental and equally infinite domains emerge, not one.

Thus, two observationally absolute singularisms are found. The first is the vertical collection of numbers between zero and one, and the second is the collection of all numbers transformed by the reversal of property, and also common to the limit specified for the domain. The dualism is formed by the nonrational relationship of these two infinite sets for the same domain. This is the same construction as found in the Russell set (discussed below). The set of all real numbers between zero and one is both a member of itself in first construction and not a member of itself in the second construction. The relationship between the constructions is nonrational.

3. Paradox in set theory

The Russell paradox is the generalized statement in the formalism of language for the structure of paradox, the *set of all sets that are not members of themselves*. It is the set of all sets for the named property, the property of not being the named property. The Russell set, called *R*, can be visualized geometrically as having two possible locations. Since *R* is not a member of itself, it must be excluded from its own domain. However if *R* is excluded, then it should be placed in its own domain as it meets the criteria of membership (stated by property) for the domain.[2] Thus even though the property of the set is clearly defined, it is not possible to limit the location of the set to the singular universe of the set of all sets. Rather, two possible well-defined (geometric) locations for the Russell set are found: It is a member of the space (contained in the limit of this universe) and it is not a member of the space (not contained in the limit of this universe). The very attribute that is used to assign location to the set prevents clear placement of it.

[2] See Penrose, 1991: pp. 99–103.

In both Cantor's argument and the Russell set, reversing a property of membership creates a paradoxical structure, and the set is then not a member of itself. In Cantor's argument, the property of reference used was the property of oneness for the real numbers between zero and one, and for *R*, the property of reference is all properties. Thus, Cantor's argument is a limited case of *R*. In both cases, the particular structure is not contained in its own rational boundary condition, and this is paradoxical. The important point to carry forward is that reversal of property for an infinitely specified domain produces a dualism that is larger than can be singularly contained.

The dualism of locations for the Russell set is in a paradoxical superposition. It is important to understand that the Russell set is perfectly rational in the sense that two isolated rational locations can be identified. The problem of paradox arises only when the relationship between these two locations is considered. The property of the locations establishes that they are not members of themselves. The context of superposition for the dualism of *R* is strongly linked to the nonlocal structure of the parts found in a quantum mechanical state. The Russell set can be viewed as the fundamental quantum mechanical object stated in the formalism of language. Its property is nonlocal for its own domain.

4. Paradox in the structure of axioms

In the above examples, the problem of establishing the limits that apply for any domain refers to the nature of location as defined by property. The same problem can be identified when it comes to limiting the axioms that apply to such structures. Roger Penrose documents the attempts to resolve this issue. Both Russell and Whitehead, and later David Hilbert, tried to produce schemes in which the mathematical structure of axioms could be clearly defined without contradiction. However, all attempts failed.[3] It was finally left to Kurt Gödel to decisively end the argument. In the words of Penrose, Gödel indisputably established that, ". . . no formal system of

[3] Penrose, 1991: p. 101.

sound mathematical rules can ever suffice, even in principle, to establish all the true propositions of ordinary arithmetic."[4]

In other words, Gödel proved that certain truths can be known to be true within an existing system of axioms, yet when the axioms are applied to these truths, the truths cannot be proved or disproved, and this is paradoxical. In the final analysis, if the members of a domain cannot be singularly contained in an infinite limit (as found in the Cantor and Russell arguments), then it is reasonable that the axioms, which establish the limit of relationships that apply to such domains, will also not be singularly contained. The totality of truths known to apply to the domain cannot be known to be true under a singular system of axioms. Rather, an absolute dualism exists for the axioms of the domain.

Drawing on the "slightly later ideas of Alan Turing"[5], Penrose provides a simplified version of Gödel's argument in his book <u>Shadows of the Mind</u>. To summarize Penrose's argument, two sets of computations are defined for a Universal Turing machine. The first set <u>stops</u>, and the second set <u>does not stop</u>. Thus, the property of stopping and not stopping is reversed across the arguments. Penrose then proceeds to show, by substitution of the terms in one of the arguments, that both sets of computations are, in fact, the same argument. Thus, the sets of computations that give different results (stopping and not stopping) are, in fact, the same set, and this is paradoxical.

Note: A *Turing machine* is constructed as a thought experiment in which a computer is capable of performing a selected infinite set of calculations. At the completion of any given infinite set, the machine may have either stopped or not stopped for conclusion relative to the question that is the basis of the infinite set of calculations.

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[4] Penrose, 1994: Ch. 2.1.

[5] Ibid.: Ch. 2.5.

The same fundamental structural reversal of property for a domain applies to Gödel's argument as for the Cantor and Russell arguments. In Gödel's argument, that which is set in reverse is reversed in the initial statements of the argument. Specifically, the conclusion found for two sets of computations using a Universal Turing machine is reversed. Because a Universal Turing machine is used, the sets of computations are known to go to completion and the structure defined for drawing conclusion is infinite. Secondly, the two sets of computations represent a fundamental dualism since their conclusions are reversed (stopping and not stopping). Finally, the reversed conclusions of this dualism are linked to each other to show that the domain they form is singular, i.e. that this dualism is the same set of computations.

All of the above arguments combine paradoxical structures in a common domain. Only the Impressionist Theory of Everything (IToE) can explain them. In each argument, a domain has been constructed in which a fundamental singularism is also a fundamental dualism. The mechanism that creates this Impressionist structure of singularism/dualism is the paradoxical reversal of property.

Note: This same general format, that the two sets of axioms are paradoxical and cannot be rationally derived from each other, is also found in the relationship between quantum mechanical and classical constructions. A structural reversal as defined under IToE is found. Specifically, the system of axioms in quantum mechanical structures is based on nondistributive logic, and the system of axioms in classical structures is based on distributive logic. For a discussion of the difference between distributive and nondistributive logic see Herbert, 1985: p. 179 (Ch. 10).

5. Conclusion

The rational response when paradoxical structures appear in fundamental theoretic arguments is to look to increasingly complex explanations for their resolution. When this strategy fails, the next possible step is to categorically and arbitrarily rule paradox inadmissible because no useful conclusion is considered possible. Rational argument for the sake of rational argument becomes the paramount objective, yet this stance is not justifiable based on the evidence.

The alternative to this dilemma, which has not been properly explored is that theoretic arguments such as those discussed in this chapter reveal a fundamental truth about the nature of the universe. For both theory and empiricism, when observationally absolute boundaries for the nature of property are approached, conclusion is not possible because of the natural mechanism of paradox. Beyond these boundaries to observability and rationalism, there always exists a second domain that is not accounted for, and this contradiction for the specification of infinity is nonresolvable. Although the mechanism of paradox cannot be rationally resolved, it can be rationally interpreted under the Impressionist Theory of Everything (IToE).

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1. Herbert, Nick. 1985. Quantum Reality, New York: Doubleday.

2. Penrose, Roger. 1991. The Emperor's New Mind, New York: Penguin Books.

3. Penrose, Roger. 1994. Shadows of the Mind, Oxford: Oxford University Press.

