The Theory Of *Murchanas* and *Murchana* Computation

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Overview

In this paper we briefly introduce a mathematical model of Indian music. We then discuss several mathematical properties of the *murchana* process [i.e., the process of deriving scales from a parent scale] that arise from this model, as well as methods for the computation of *murchanas* of *rags* [melodic modes]. Finally, we end the paper by illustrating some of the mathematical properties of Rag Malkauns from the point of view of *murchanas*.

Introduction

One of the unfortunate aspects of Indian music theory is that its largely mathematical character has been eschewed by musicologists in favor of its more romantic aspects. While it may be interesting to delve into the *ras* [mood] of a *rag*, or the emotive characterization of *shrutis* [microtonal inflections], or even the so-called appropriate *samay* [time of day] for the rendition of particular *rags*, such endeavors are, at best, error-plagued and, at worst, completely subjective. In this sense, it is perhaps more constructive to consider some of the more scientific aspects of Indian music.

Since mathematics, like music, is as much an art as it is a science, it is debatably the best tool that we can use to approach the scientific side of music. The purpose of this paper, then, is threefold. First, we intend to introduce the reader to the idea of a mathematical model of Indian music. Secondly, we will apply the constructed model to show some mathematical properties of the *murchana* transform. In the process of discussing *murchanas* we will also discover a number of very practical ways of calculating the *murchanas* of any given *rag*. Finally, we will illustrate how this model reveals the mathematical beauty of a particular *rag*, Rag Malkauns.

Sequences and *rags*

Let us begin by realizing that a *saptak* [the octave] contains exactly twelve *swarsthans* [pitch positions]. These *sthans* are given in Hindustani (i.e. North Indian) music by a kind of "moveable doh" system: Sa, *komal* [flat] Re, Re, *komal* Ga, Ga, Ma, *tivr* [sharp] Ma, Pa, *komal* Dha, Dha, *komal* Ni, Ni. These may best be notated as

S r R g G m M P d D n N $\dot{S}$ [1]

We used the term *swarsthan* instead of *swar* [note] because enharmonic spelling is used occasionally in Hindustani music and quite prominently in Karnatak music. For this reason, it is best to think of these frequencies as *sthans* instead of *swars* themselves. However, for our purposes, it is sufficient to use *swar* and *swarsthan* interchangeably, without differentiating between them. Now, consider an enumeration scheme, such that we associate 0 with Sa, 1 with re and so on, until 11 corresponds to Ni. Hence, instead of the series of *swars* given by Sa, re...Ni, we can represent the *swars* of the saptak more compactly as the sequence of integers 0, 1...11.

Now, it becomes intuitively obvious that given a sequence of *swars*, we can produce a sequence of integers, all between 0 and 11, that can be associated with the sequence of *swars* uniquely. In particular, we note that any *rag* can be characterized by two unique sequences of finite length; the *aroh* and *avroh* [ascending and descending scales]. For example, if we look at Rag Malkauns, which we will study in more detail later, we have the following *aroh* and *avroh* sequences:

S g m d n $\dot{S}$
$\dot{S}$ n d m g S

By using our enumeration scheme, these sequences can be rewritten as [0, 3, 5, 8, 10] and [0, 10, 8, 5, 3]. We do not bother to add on a 0 for the last Sa of each sequence, since we include Sa at the beginning of each sequence. Hence it is already present in the sequence.

Since any model is typically limited in many ways, we must put some limits on our model as well. To do so we restrict the class of *rags* that we will study. Unfortunately the set of all *rags* cannot be dealt with by one model, since structural diversity is so common among them. For our purposes, let us assume that a *rag* has the following characteristics:

1 Each *rag* corresponds to a unique sequence
2 The length of any given *rag* sequence must be at least five *swars*
3 The sequence of the *rag* must contain 0
4 No *rag* with chromatic scales will be considered.
5 The *swars* of the *rag* scale are fairly well distributed across the *saptak*.

Before proceeding, let us consider the implications of each restriction we have placed on our model. First, we demand that each *rag* corresponds to a unique sequence, *not two unique sequences!* Hence, the *aroh* must be the identical reverse of the *avroh* and vice versa. Our model does not accommodate *rags* like Bhimpalasi whose ascent is different from its descent. Next we demand that there be at least [five *swars*] in the sequence of the *rag*. Therefore, *rags* with fewer than five [*swars*], like Malashree, cannot be considered. We also demand that Sa be present in the sequence of any *rag*. Therefore, a *rag* like Yaman must have its *aroh* modified to Sa, Re, Ga [0, 2, 4] instead of Ni, Re, Ga [11, 2, 4]. Since the majority of *rags* (with exceptions like Nand and Lalit) are diatonic in nature, we place the fourth restriction on our model. Finally, we require that the *swars* be somewhat evenly scattered through the saptak. This idea is somewhat vague. But, its purpose is to avoid labelling sequences like [0,1,3,5,7] as *rags* (i.e., with no *swars* in the upper tetrachord).

The result of the formulation of the model that we have described in this section is that we can now treat a *rag* as a mathematical object, corresponding to a sequence of integers. We cannot and do not wish to deny that such restrictions severely limit the scope of our work. But the point is to limit ourselves to a fairly small group of *rags* at first, and then to branch outward and consider more structurally complex *rags*. We leave, for future work, consideration of *rags* which violate the model that we have described here.

A brief overview of the *murchana* transform

We now come to the main discussion of this paper, the *murchana* transform. In an excellent treatise on the *murchana*, Sharad Gadre (1993) discusses its definition and gives a very comprehensive mapping of *rags* to their *murchanas*. In this, we give a brief definition of a *murchana* before delving into the mathematics of this operation on *rags*.

When we hear a piece of Indian music we typically identify one *swar* very quickly, namely, Sa. This identification allows us to create in our minds a reference point from which we can identify the remaining *swars* of the *rag*. Whether we execute this process on a conscious or subconscious level is immaterial. The point is that Sa usually is the first to be identified in a rendition, and it serves to help us fix other pitches.

Now, what would happen if, during a performance, the musician were to abruptly switch the Sa to some other frequency? After the initial jarring effect, the result would be that we would tune into the new Sa and then map out the remaining *swars* of the *rag* in relation to it. So, although the musician would be using the same set of frequencies in his

Medallion relief from Amaravati, 2nd - 3rd century A.D.

rendition, changing the position of the Sa would cause us to recognize a new set of *swars* being used. Thus, effectively a completely different *rag* would emerge. This new *rag* is called the *murchana* of the original *rag*. Why would a new set of *swars* arise? Because the reference point has changed.

Let us illustrate this idea with an example. Suppose we are listening to Rag Abhogi whose *aroh* and *avroh* is

S R g m D Ṡ
Ṡ D m g R S

We wish to relocate Sa on Ma. If we place Sa where Ma used to be, the original Dha of Abhogi must change its name to Ga because the interval Ma-Dha is equal to the interval Sa-Ga (i.e. the original note moves up five chromatic steps, or a perfect fourth]. The original Sa now becomes Pa, Re becomes Dha, and *komal* Ga becomes *komal* Ni and so on to produce the following *murchana*

S G P D n Ṡ
Ṡ n D P G S

This is the *aroh-avroh* of Rag Kalavati. So we find that Kalavati is a *murchana* of Abhogi. But is Abhogi a *murchana* of Kalavati? Yes, because when we superimpose the scale of Abhogi on the scale of Kalavati we find that Abhogi's Sa maps to Kalavati's Pa. Hence, if one derives a *murchana* of Kalavati with Pa as the new Sa, it stands to reason we will return to the *swars* of Abhogi.

Sequences and the *murchana* transform

The calculation of *murchanas* can be complicated and prone to error, but by using our method of converting *rags* to equivalent sequences of integers we can reduce the task to a simple exercise of modular arithmetic. It may be appropriate to give a very cursory introduction to modular arithmetic here. For further details, beyond the scope of this paper, the reader is referred to any elementary text on Number Theory.

Consider our original enumeration scheme of associating the integers from 0 to 11 with *swars*. For the next part of our discussion we have to add and subtract these integers to receive new integers. But the problem is that a mathematical statement such as $11 + 1 = 12$ makes no sense in our model. There is no *swar* that corresponds to the integer 12. Similarly, what does -1 mean in the statement $0 - 1 = -1$? To resolve this dilemma, let us consider what + and – really mean to us in the context of our model. When we say $11 + 1$, the suggestion is that we are considering movement one step above the enumerated *swar* 11.

But 11 is Ni. One step above Ni is Sa, enumerated by 0 and not 12. Therefore, 11 + 1 = 0. Similarly, if we consider 0 – 1, the suggestion is that we are going to move one step down from the enumerated *swar* 0 (Sa). But this step down gives us the *swar* Ni. Therefore, 0 – 1 = 11. This method of calculation usually seems strange in the beginning. But, the reasoning behind the sums is relatively simple. These additions and subtractions are merely moving us through a circle of *swars*. If the enumerated *swars* are listed in increasing order from 0 to 11 clockwise around the circle, an addition of x units entails moving x units in the clockwise direction around the circle. A subtraction of x units moves us x units in the counterclockwise direction around the circle of *swars*.

Another manner in which these calculations can be carried out is to take the sum in the usual manner and then divide it by 12. The remainder of this division is the result we need. For example, 11 + 1 = 12. 12 / 12 = 1, with remainder 0. Therefore, 11 + 1 = 0. Similarly, 9 + 7 = 16. 16 / 12 = 1, with remainder 4. Therefore, 9 + 7 = 4.

So, what is the use of all this mathematical manipulation? Let us go back to our examples of Abhogi and Kalavati. Abhogi, in our model, can be represented by the sequence [0, 2, 3, 5, 9]. Now, suppose we want to calculate the same *murchana* that maps Abhogi to Kalavati. Ma is enumerated by 5 in Abhogi. But, we want to transform the sequence so that instead of 5, the same frequency will now map to 0. But this is equivalent to solving the equation $5 + x = 0$. It is not difficult to see that $x = 7$ will do the trick for us. Therefore, all we really need to do is add 7 to each element of the sequence of Abhogi, to get the sequence for the Ma *murchana* of Abhogi, thus

0 + 7 = 7
2 + 7 = 9
3 + 7 = 10
5 + 7 = 0
9 + 7 = 4

Therefore, the new sequence is given by [7, 9, 10, 0, 4]. After adjusting the order of the integers we get [0, 4, 7, 9, 10]. Now, if we convert back from enumerated *swars* to the *swar* names, we get the following *aroh-avroh*

S G P D n Ṡ
Ṡ n D P G S

As we predicted, this is precisely the aroh-avroh for Rag Kalavati. Let us then generalize this procedure for any given rag. Suppose $R = [0, r_1, \ldots, r_{n-1}]$ is the sequence characterizing a particular *rag*. Let $M_i(R)$ denote the sequence of the murchana of the rag determined by equating r_i with Sa, for all $i = 0, 1, \ldots, n-1$. In our system of moving around the *swar* circle we use the notation $x + y \equiv z(mod 12)$, where z is the remainder of the division of $x + y$ by 12. Therefore, if $r_i + \rho \equiv 0(mod 12)$, then $M_i(R) = [\rho,(r_1 + \rho)mod 12,(r_2 + \rho) mod 12,\ldots, (r_{n-1} + \rho)mod 12]$. A point that we do not prove here, but which is true nevertheless, is that this sequence is equivalent to the sequence $[\rho,r_1 + \rho,r_2 + \rho,\ldots,r_{i-1} + \rho,0,(r_{i+1} + \rho)mod 12,\ldots,(r_{n-1} + \rho)mod 12]$.

The point of this exercise is that calculating the *murchana* of a *rag* using our method is considerably easier and more reliable than methods based on transposing intervals. Moreover, our calculations took only a few lines. We now prove two very simple theorems about the *murchana* transform.

> **Theorem:** If R is the sequence of a rag, and r is a *swar* that does not appear in R, then the transform of R using r is not the sequence of a *rag*.
>
> **Proof:** If we equate r with Sa, it follows that 0 will not appear in the murchana of R. But, in our model, every sequence must contain 0. Therefore, the resulting murchana cannot be a rag. *QED*.

Before proceeding we need to introduce a simple definition for a projection function. We give the formal mathematical definition below.

> **Definition:** Given a sequence $R = [0,r_1,\ldots,r_{n-1}]$, define the projection function π_i as a function from sequences to the integers from 0 to 11, such that $\pi_i\ (R) = r_i$, for all $i = 0,\ 1,\ldots,n\text{-}1$.

Informally, the function π_i, simply takes a sequence and returns the integer found in the *i*th place of the sequence. So, for example, if M is the sequence of Malkauns, $\pi_0(M) = 0$, $\pi_1(M) = 3$, and $\pi_2(M) = 5$ and so on. Given this definition, we can now pin down exactly those *murchanas* of *rags* which are not *rags* themselves.

> **Theorem:** Let R be the sequence of a rag such that $\pi_i(R) - \pi_{i+1}(R) = 1$. Then, if $\pi_i(M_j(R))$ is a member of the set {1,3,5,8,10}, then $M_j(R)$ cannot be a *rag*.
>
> **Proof:** If $\pi_i(M_j(R))$ is a member of the given set, then it follows that $\pi_{i+1}(M_j(R)) = \pi_i(M_j(R)) + 1$ which implies that the sequence of the murchana contains one of the sub-sequences in the set {[1,2],[3,4],[5,6],[8,9],[10,11]}. But if that is the case, then the murchana is a chromatic scale. Therefore, it cannot be a *rag*. *QED*.

The *murchana* transform matrix

So far, we have suggested one fairly straightforward manner in which we can calculate the *murchanas* of *rags*. However, it does involve some slightly unfamiliar bits of mathematics which may cause non-mathematicians to be ill-at-ease. Perhaps an even more straightforward way can be devised. What if we were to perform the calculations suggested in the last sections for the non-mathematicians? This suggestion is not as ridiculous as it seems at first. We have, after all, twelve possible *swars*, each of which can be transformed using twelve possible *murchanas*. For example, if we want to transform the enumerated *swar* 5 by using the enumerated *swar* 8 as the new Sa, we simply have to realize that $8 + 4 \equiv 0(mod 12)$. Hence, to transform 5 we add 4 to it to get 9. Therefore, taking the *murchana* of a *rag* sequence by equating 8 with 0, will force 5, if present in the scale, to map to 9. But we can perform this process for each and every possibility, resulting in a 12 x 12 matrix of the following form.

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	11	0	1	2	3	4	5	6	7	8	9	10
2	10	11	0	1	2	3	4	5	6	7	8	9
3	9	10	11	0	1	2	3	4	5	6	7	8
4	8	9	10	11	0	1	2	3	4	5	6	7
5	7	8	9	10	11	0	1	2	3	4	5	6
6	6	7	8	9	10	11	0	1	2	3	4	5
7	5	6	7	8	9	10	11	0	1	2	3	4
8	4	5	6	7	8	9	10	11	0	1	2	3
9	3	4	5	6	7	8	9	10	11	0	1	2
10	2	3	4	5	6	7	8	9	10	11	0	1
11	1	2	3	4	5	6	7	8	9	10	11	0

In order to use this matrix we select a row. The leftmost number outside of the parenthesis indicates the enumerated *swar* that we will use as the new Sa. The column numbers along the top of the matrix and outside of the parenthesis indicate the *swars* being transformed. For example, if we want to know what *swar* 7 will transform to under a *murchana* that equates 2 with the Sa, we simply go to the entry in row 2, column 7 of the matrix. We discover that this number is 5. Now, consider our example of Abhogi and Kalavati again. Abhogi is given by the sequence [0, 2, 3, 5, 9]. If we want to transform the sequence using Ma as the new Sa, we will look in row 5 of the matrix. We see that 0 maps to 7, 2 to 9, 3 to 10, 5 to 0 and 9 to 4. Therefore the sequence for the transform is [7, 9, 10, 0, 4], or more appropriately [0, 4, 7, 9, 10] , which is once again the sequence for Kalavati.

As the above example illustrates, the matrix, which we will call the *murchana* transform matrix (MTM), helps us calculate the *murchana* of a *rag* even faster. We simply look in the matrix for the correct mapping to figure out the *murchana*. There is no mathematical calculation involved at all. In addition to being a very powerful tool in *murchana* calculation, the *murchana* transform Matrix has very beautiful mathematical properties as well. We will look at some of these properties when we discuss Rag Malkauns later. However, for the moment, we make a curious discovery about the matrix. The MTM can be abbreviated into a simple matrix polynomial. Define the matrix C, as follows

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Thus, C is exactly the matrix with entries $c_{i,j} = 1$ if $j \equiv (i + 1)(mod 12)$. Otherwise $c_{i,j} = 0$. Then the MTM can be written as the following polynomial in terms of C.

$$\sum_{j=0}^{11} jC^j$$

Successive matrix multiplications of C by itself simply shift the diagonal of 1's up one diagonal each time, thus allowing us to formulate this polynomial form of the matrix. The main advantage of this is that it can be retained in memory more easily than the bulky 12 x 12 matrix itself. If the polynomial is memorized, it is a simple mathematical step to formulate the matrix itself from the formula.

Binary representation of *rags*

There is yet another very easy and efficient way of calculating the *murchana* of a *rag*. This method was suggested by P. Sriram (1990). Consider the following fact: each of the twelve *swars* is either in the sequence of a *rag*, or it is not. Hence, the presence or absence of a *swar* can be denoted by a binary digit (bit). If the bit is 1 for a *swar*, then it is present in the sequence. If it is 0, then it is not present. In this manner, we can represent any *rag* by a sequence of twelve bits (one for each *swar*). For example, Malkauns can be represented by the string of bits 100101001010, where the first bit represents Sa, the second *komal* Re and so on. Since *komal* Re is not present in Malkauns, the corresponding bit is set to 0. Since Sa is present, the corresponding bit is set to 1.

In order to compute the *murchana* of a *rag*, we simply choose the position which will correspond to the new Sa and copy down the bit string of the original *rag* from that point on till the end of the bit string. We then append to this string, the substring from the beginning of the bit string of the original *rag* to the position with the new Sa. We can illustrate this process, again, with our Abhogi-Kalavati example. The bit string for Abhogi is given by 101101000100. Ma(S) corresponds to the fourth 1 in this string. Hence, we begin by copying down the Abhogi substring from the fourth 1 to the end of the string — 1000100. Next we find the substring from the beginning of the Abhogi string to the position of the new Sa, which, in our case is the fourth 1 in the string. This gives us the substring 10110. Now, we simply append this substring to the substring from the new Sa to the end of the Abhogi string, giving us $< 1000100 >< 10110 > \Rightarrow 100010010110$. The first 1 in this string corresponds to Sa, the second to Ga, the third to Pa, the fourth to Dha and the fifth to *komal* Ni, giving us the *aroh-avroh*

S G P D n Ṡ
Ṡ n D P G S

which, as we predicted is exactly the scale of Kalavati. I have given proofs of a number of interesting properties of these binary strings elsewhere (Mahalanabis 1995b).

Special properties of Rag Malkauns and the MTM

We conclude the paper by noting the properties of Rag Malkauns from the point of view of *murchanas*. Malkauns is mathematically one of the most elegant *rags*, as we shall point out in this section. First, we note that for the majority of *rags*, as documented by Gadre (1993), not all *murchanas* produce new *rags*. For example, if we look at Rag Yaman, which is represented by the sequence [0, 2, 4, 6, 7, 9, 11] we find that the *murchana* using *tivr* Ma as the new Sa gives the sequence [0, 1, 3, 5, 6, 8, 10]. But this is clearly not a *rag* in our model because it is chromatic. However, Malkauns is very special in this respect, because every *murchana* transform of the *rag* gives a new *rag*. If we remove the rows and columns of the MTM that represent *swars* not found in Malkauns we get a reduced MTM, given below

$$\begin{array}{c} \\ 0 \\ 3 \\ 5 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 3 & 5 & 8 & 10 \end{array} \\ \begin{pmatrix} 0 & 3 & 5 & 8 & 10 \\ 9 & 0 & 2 & 5 & 7 \\ 7 & 10 & 0 & 3 & 5 \\ 4 & 7 & 9 & 0 & 2 \\ 2 & 5 & 7 & 10 & 0 \end{pmatrix} \end{array}$$

It should be noted that such a matrix can be constructed for any *rag*. The advantage of such a matrix is that each row represents a *murchana* of the *rag*. In this case, the first row is simply the sequence for Malkauns. The second row is the sequence of Rag Durga. The third row is the sequence for Rag Dhani. The fourth row is the sequence for the Rag Bhoop. The fifth row is the sequence for the Rag Megh. This, then, is the beauty of Malkauns; each row of the reduced MTM for Malkauns identifies a unique *rag* that Malkauns maps to under a *murchana* transform. As we said earlier, this is certainly not the case for most *rags*.

In Mahalanabis (1995a) I introduced an operator on *rags*, $\triangle$, which, when applied, gives the set of *rags* which differ from the original *rag* in exactly one location in its sequence. This operator can be applied iteratively to sets as well as *rags*. In general, if R is the sequence of a *rag*, $\triangle^n R$ is the set of *rags* that differs from the original *rag* in exactly n positions in its sequence. We note that the *murchanas* of Malkauns form a chain under this operator, in the sense that

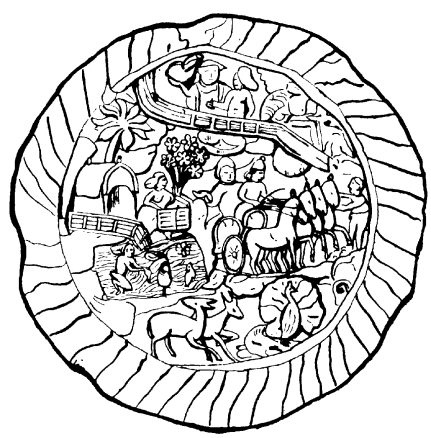
Malkauns	=	[0, 3, 5, 8, 10]	= Malkauns
Dhani	=	[0, 3, 5, 7, 10]	$\in \triangle$ (Malkauns)
Megh	=	[0, 2, 5, 7, 10]	$\in \triangle^2$ (Malkauns)
Durga	=	[0, 2, 5, 7, 9]	$\in \triangle^3$ (Malkauns)
Bhoop	=	[0, 2, 4, 7, 9]	$\in \triangle^4$ (Malkauns)

where $\in$ indicates that the *rag* is a member of the given set.

We now turn back to the MTM. When we look at this matrix carefully, an interesting property becomes apparent. If $m_{i,j}$ is the entry in the *i*th row and *j*th column, $m_{i,j} + m_{j,i} \equiv 0(mod 12)$. Such types of matrices are referred to as skew symmetric. We now prove a general theorem regarding MTM's of all *rags*.

Theorem: The reduced MTM for any given *rag* is skew symmetric.

Proof: In order to write out the MTM of any *rag*, we must delete the rows and columns of the MTM which do not correspond to *swars* in the *rag* sequence. Now, suppose $m_{i,j}$ is in the reduced MTM. It follows that row i and column j were not deleted, which implies that row j and column i were also not deleted. Hence $m_{j,i}$ must also be in the reduced MTM. But we know that $m_{i,j} + m_{j,i} \equiv 0(mod 12)$. Thus, the reduced MTM must be skew symmetric. *QED*.



Terracotta plaque, Bhita, 1st century B.C.

Conclusion

In this paper, we have discussed a number of simple methods of calculating the *murchana* transforms of *rags*. In addition we have shown a number of mathematical properties of the *murchana* transform, the MTM, and the reduced MTM. Finally, we have briefly discussed the mathematical properties of Rag Malkauns. There are still a number of interesting open problems: what happens to the theory presented in this paper when we bring in *rags* which violate our model? What about cases where the *rags*' melodic movements are extremely *vakra* [oblique, zig-zag] in character? What happens when we take the *murchanas* of these *rags*? Are the resulting sequences valid *rags*? Further, we have left open the problem of finding other *rags*, like Malkauns, which, when transformed using each and every *swar* in its sequence produces a new *rag*. Again, we do not know if there are other *rags* like Malkauns which produce a chain under the $\triangle$ operation. Obviously this problem is even more difficult to solve than the first Malkauns problem, since it requires not only that every *murchana* be a *rag*, but also that it be a link in the chain. Clearly, there is much scope for more work in the mathematical aspects of Indian music. Hopefully, this paper will be able to persuade others to pursue this area of research in the future.

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Notes

1 Dots above notes signify upper octave (and dots below signify the lower octave).

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